Algebra I Exam
2009 University of Houston Math Contest

Name: _____

School: _____

Please read the questions carefully and give a clear indication of your answer on each question.

There is no penalty for guessing.

Judges will use written comments and/or calculations to settle ties.
Good luck.

Name: _____

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University of Houston
High School Contest – Spring 2009
Algebra I Test

Exam Time : 1 hour

No calculator allowed.

Write the letter (a, b, c, d or e) corresponding to your answer in the table below.

There is no penalty for guessing.

In the event of a tie, students' work will be used to determine the winner – so show your work clearly on your test or scratch paper.

Problem	Answer	Problem	Answer
1		14	
2		15	
3		16	
4		17	
5		18	
6		19	
7		20	
8		21	
9		22	
10		23	
11		24	
12		25	
13			

1. Which of the following, if any, is/are true.

I. $10y - x^2 = -1$ is not a function.

II. If the slope of line L_1 is $m_1 = -7$ and L_1 is perpendicular to line L_2 then $m_2 = \dfrac{1}{7}$.

III. The range of the function $g(x) = \dfrac{-5}{3}x - \dfrac{1}{2}$ is the set: $\left\{y \mid y \leq -\dfrac{1}{2}\right\}$.

IV. The set $\{(-1, 2), (2, -4), (5, 6), (-1, 7), (-4, 4)\}$ is a relation that does not define a function.

V. Let $h(x) = x^2 + 1$. The function has exactly 2 real zeros.

a. I, III, V

b. II, IV

c. None are true

d. I, III

e. II, IV, V

2. You take an average of x hours to process a work form, whereas your team member Coreen's average is 1 hour longer. Which expression represents the number of work forms that you and Coreen can process in 40 hours?

a. $80x^2 + 40x$

b. $40x + 20$

c. $\dfrac{80x + 40}{x^2 + x}$

d. $80x + 40$

e. $\dfrac{160x + 80}{x^2 + x}$

3. Let $P(x, y)$ be a point on the graph of $f(x) = x^2 - 5$. Express the distance from P to the origin as a function of x.

a. $d(x) = \sqrt{x^4 - 9x^2 + 25}$

b. $d(x) = \sqrt{x^2 - 25}$

c. $d(x) = \sqrt{x^4 - 2x^3 + 9x^2 + 10x + 25}$

d. $d(x) = \sqrt{x^4 - 625}$

e. $d(x) = \sqrt{x^4 + x^2 + 25}$

4. Solve for x.

$$\dfrac{4^{3x}}{\sqrt[3]{4}} - 8 = 0$$

a. $x = \dfrac{7}{18}$

b. $x = \dfrac{25}{9}$

c. $x = \dfrac{7}{9}$

d. $x = \dfrac{11}{18}$

e. $x = \dfrac{5}{9}$

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School: _____

The following problem is for problems 5 and 6.

A company car was purchased for $25,000. The car will be depreciated linearly over 7 years. The value of the car at 7 years will be $4,000.

5. Sketch the graph that represents this model. What is the domain?

a. All real numbers
b. $[0, 7]$
c. $[7, 4,000]$
d. $[0, 25,000]$
e. $[4,000, 25,000]$

6. Find the value of the car after 3 years.

a. $3,000
b. $21,000
c. $7,000
d. $9,000
e. $16,000

7. A rectangle with width $2x + 5$ feet has an area of $2x^4 + 15x^3 + 7x^2 - 135x - 225$ square feet. Determine the polynomial that represents the perimeter of the rectangle.

a. $6x^3 + 10$
b. $2x^3 + 20x^2 + 2x - 120$
c. $x^3 + 10x^2 - x - 65$
d. $2x^3 + 10x^2 - 14x - 80$
e. $2x^3 + 10x^2 + 22x + 180$

8. Multiply. $(2x^{a-1} - 3y^{b+5})(2x^{2a-3} - 3y^{4-b})$

a. $4x^{3a-4} - 12x^{3a-4}y^9 + 9y^9$
b. $4x^{3a-4} + 9y^9$
c. $4x^{3a-4} - 6x^{a-1}y^{-b+4} - 6x^{2a-3}y^{b+5} + 9y^9$
d. $4x^{2a^2-5a+3} + 9y^{-b^2-b+20}$
e. $4x^{3a-4} - 9y^9$

9. The points $(1, y)$ and $(4, -2)$ are on line L_1 and the points $(2, 8)$ and $(-7, y + 4)$ are on line L_2. If L_1 is parallel to L_2, what is the value of y?

a. -5
b. 3
c. -2
d. 4
e. -23

10. Which of the following is/are false?
I. The point $(-3, 5)$ satisfies the equation $13x + 6y + 9 = 0$.
II. The x-intercept of a line is a and the y-intercept of the same line is b. Assume $a \neq 0$ and $b \neq 0$. Then the equation of the line may be written in the form: $\dfrac{x}{a} + \dfrac{y}{b} = 1$

III. Given the following system of inequalities: $\begin{array}{l} x + 2y \geq 3 \\ 2x + 4y \leq -2 \end{array}$.

The solution set is all real numbers.
IV. The ordered pair $(-3, 2)$ is in the solution set for $y < x + 5$.

V. The y-intercept of the line $Dx + Ey + F = 0$, with $D \neq 0$, is $\left(0, \dfrac{-F}{D} \right)$.

a. I, II, IV
b. III, IV, V
c. III, V
d. I, II
e. II, III

11. An aerobics instructor gives a class both in the morning and afternoon for 3 days, off for 1 day, gives a class in the morning only for 2 days, off for 2 days, and then her schedule begins again. If she is on her 1 day off part of the schedule, determine what she will be doing 127 days from today.

a. Giving a class in the morning only.
b. Not enough information is given to determine the answer.
c. Off on the 1 day part of the schedule.
d. Giving a class both in the morning and afternoon.
e. Off on the 2 days part of the schedule.

12. A company makes surge protectors. The supply equation is $S = 5000 + 200x$ and the demand equation is $D = 9500 - 100x$, where x represents the price (in dollars) of a surge protector. The price at which supply and demand are equal is called the equilibrium price. What will be the demand at the equilibrium price?

a. 5,000
b. 15
c. 14,000
d. 45
e. 8,000

13. One factor of $x^2 + 16x - 4 + 64 - y^2 + 4y$ is.

a. $x + y + 6$
b. $x - y$
c. y^2
d. $y + 4$
e. x^2

14. Simplify. $\dfrac{\sqrt{x}}{\sqrt{x} - \sqrt{x-3}}$

a. $\dfrac{x + \sqrt{x^2 - 3}}{3}$

b. $\dfrac{x + \sqrt{x^2 - 3x}}{3}$

c. $\dfrac{x^2 - 2x}{3}$

d. $\dfrac{-x - \sqrt{x^2 - 3x}}{3}$

e. $\dfrac{-x^2 - 2x}{3}$

15. Simplify.
$$\frac{\dfrac{x}{1-\dfrac{x}{2+2x}}-2x}{\dfrac{2x}{5x-2}-3}$$

a. $\dfrac{(2+x)(5x-2)}{2x(13x-6)}$

b. $\dfrac{(x^2+2x-4)(5x-2)}{2(-13x+6)}$

c. $\dfrac{2x(5x-2)}{(2+x)(13x-6)}$

d. $\dfrac{(2x-4)(5x-2)}{2(-13x+6)}$

e. $\dfrac{(2x-4)(5x-2)}{(2+x)(13x-6)}$

16. One of the following expressions has a value different from the other expressions. What is this different value?

I. $(-1)^{-12}$

II. $-\left(\dfrac{10}{23}\right)^{0}$

III. $-2^{-1}-2^{-1}$

IV. $-1^{-1}\cdot1^{-1}$

V. $-(1)^{\frac{1}{4}}$

a. -2
b. -1
c. 0
d. 1
e. 2

17. Katerina has math test scores of 95, 87 and 86. She has one more test to take. All exams are equally weighted. Determine what scores on the last test will allow her to get an A in the course. (An "A" is 90% or above.)

a. It is not possible for Katerina to get an A in the course.
b. $x \geq 90$
c. $x \geq 80$
d. $x \geq 92$
e. $x \geq 97$

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School: _____

18. The equation $2x - 3x^{1/2} + 1 = 0$ has two solutions. The sum of the solutions is:

a. 1

b. $\dfrac{1}{2}$

c. $\dfrac{5}{4}$

d. 4

e. $\dfrac{3}{2}$

19. Araceli goes to the bank to get $10 worth of change. She requests twice as many quarters as half-dollars, twice as many dimes as quarters, three times as many nickels as dimes, and no pennies or dollars. What is the total number of dimes and nickels?

a. 80

b. 30

c. 10

d. 2

e. 5

20. A pair of fair dice is cast. If the absolute value of the difference in the face values of the two dice is determined, what is the probability that the absolute value of the difference is odd?

SECOND DIE

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

FIRST DIE

a. $\dfrac{2}{9}$

b. $\dfrac{1}{3}$

c. $\dfrac{1}{2}$

d. $\dfrac{1}{9}$

e. $\dfrac{1}{6}$

21. Given the sequence 20, 24, 28, 32, 36, ... find the 60th term.

a. 216
b. 64
c. 256
d. 60
e. 80

22. Bob has a rectangular area he wishes to enclose with 120 feet of fencing. One side of the area is along the straight portion of a river, so no fencing is needed there. Determine an equation that describes the area enclosed.

a. $A(x) = \dfrac{x^2 + 60x}{x}$

b. $A(x) = -2x^2 + x$

c. $A(x) = \dfrac{2x^2 + 120x}{x}$

d. $A(x) = -x^2 + 60x$

e. $A(x) = -2x^2 + 120x$

23. A survey of 400 people was taken. 223 indicated that they own a jet ski, 172 indicated that they own a sailboat, and 87 indicated that they do not own a jet ski or a sailboat. How many people surveyed own a jet ski and a sailboat?.

a. Not enough information is given.
b. 51
c. 5
d. 82
e. 313

24. Solve $x^2 + ax + b = 0$ for x in terms of a and b.

a. $x = -a - b$

b. $x = -\dfrac{a^2}{4} \pm \dfrac{\sqrt{-4b + a^2}}{2}$

c. $x = \dfrac{2a - 2i\sqrt{b}}{2}$

d. $x = \dfrac{a^2 - 2a - 4b}{4}$

e. $x = -\dfrac{a}{2} \pm \dfrac{\sqrt{-4b + a^2}}{2}$

25. Four sets of brothers and sisters are to be seated in a row of eight seats. In how many ways can they be seated if each pair of brother and sister is seated together?

a. 384
b. 16
c. 8
d. 40,320
e. 4

Geometry Exam
2009 University of Houston Math Contest

Name: _____

School: _____

Please read the questions carefully and give a clear indication of your answer on each question.

There is no penalty for guessing.

Judges will use written comments and/or calculations to settle ties.
Good luck.

Name: _____ School: _____

University of Houston
High School Mathematics Contest
Geometry Exam – Spring 2009

Directions:
You have 60 minutes to complete this exam. Calculators are not permitted. Choose the correct answer for each question, and write the <u>letter</u> (A, B, C, D or E) corresponding to that answer in the blank to the right of each question. There is no penalty for guessing. In the case of a tie, students' work will be used to determine the winner – so show all work clearly on either your exam or on scrap paper. Write your name and school on top of each page in the blanks provided.

Note: Geometric figures in the problems may not be drawn to scale.

1.   Find the center and radius of the circle defined by the equation $(x+2)^2 + (y-7)^2 = 9$.

   (A)   Center $(2,-7)$, Radius 3

   (B)   Center $(-2,7)$, Radius 9

   (C)   Center $(-2,7)$, Radius 3

   (D)   Center $(2,-7)$, Radius 9

   (E)   Center $(-2,7)$, Radius 81

1. _____

2.   If a circle has a circumference of 16π inches, find its area.

2. _____

   (A) 8π in^2 (B) 16π in^2 (C) 32π in^2 (D) 64π in^2 (E) 256π in^2

3.   What can you conclude about about $\triangle ABC$ and $\triangle DEF$, given that $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\angle A \cong \angle D$?

   (A)   The two triangles are similar.
   (B)   The two triangles are congruent.
   (C)   The two triangles are both scalene
   (D)   The two triangles are both acute.
   (E)   There is not enough information to draw a conclusion.

3. _____

4.   Find the ratio of the edge of a cube to the cube's diagonal.

4. _____

   (A) $\sqrt{2}$ (B) $\dfrac{\sqrt{2}}{2}$ (C) $\dfrac{1}{3}$ (D) $\sqrt{3}$ (E) $\dfrac{\sqrt{3}}{3}$

5. Given that the statement "If p, then q" is false, which of the following statements <u>must</u> always be correct?

          I.   "If not p, then not q" is false.
          II.   "If p, then not q" is true.
          III. "If q, then p" is true.
          IV. "If not q, then not p" is false.

5. _____

    (A)   I and II only
    (B)   III only
    (C)   I and III only
    (D)   IV only
    (E)   II and IV only

6. $\overline{RT}$ is an altitude of $\triangle QRS$, and $m\angle QRS = 90°$. If $ST = 5$ and $QT = 7$, find the length of $\overline{RS}$.

    (A) $\sqrt{35}$

    (B) $2\sqrt{15}$

    (C) $2\sqrt{21}$

    (D) $\sqrt{74}$

    (E) $2\sqrt{6}$
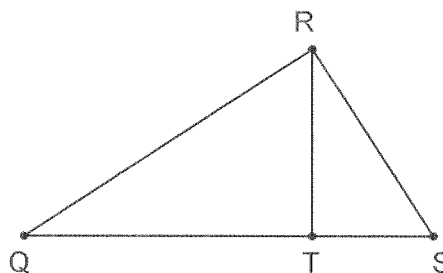
6. _____

7. A chord of length 24 cm is drawn in a circle of radius 13 cm. Find the distance from the chord to the center of the circle.

7. _____

    (A) 25 cm   (B) 11 cm   (C) 12 cm   (D) 5 cm   (E) 13 cm

8. Find the measure of an angle satisfying the following:
Six times the complement of an angle is five less than the supplement of the angle.

    (A) $25°$
    (B) $71°$
    (C) $73°$
    (D) $35°$
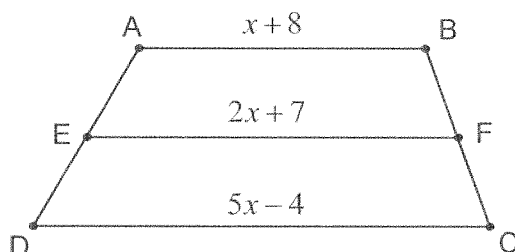   (E) $26\,\frac{2}{7}°$

8. _____

9. Isosceles triangle *TRI* has base angles *T* and *I* and median $\overline{RM}$. If the perimeter of $\triangle TRI$ is 54 inches and $TM = 4$ inches, find the length of $\overline{TR}$.

(A) 8 inches
(B) 38 inches
(C) 23 inches
(D) 25 inches
(E) 46 inches

9. _____

10. $\overline{EF}$ is the median of trapezoid *ABCD*, with measures listed as below. Find the length of $\overline{AB}$.
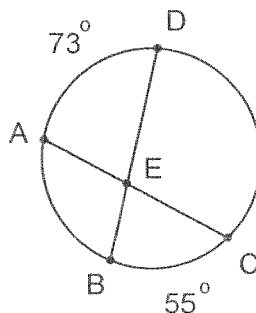
(A) 5

(B) 13

(C) $\dfrac{21}{4}$

(D) 17

(E) 21

10. _____

11. In the circle below, chords $\overline{AC}$ and $\overline{DB}$ intersect at point *E*. If $m\overparen{AD} = 73°$ and $m\overparen{BC} = 55°$, find $m\angle AED$.

(A) 36.5°
(B) 55°
(C) 64°
(D) 73°
(E) 128°

11. _____

12. Let ℓ represent the line which passes through the point $(-8, -9)$ and is perpendicular to the line $y = -\dfrac{2}{5}x + 6$. Find the *x*-intercept of ℓ.

12. _____

(A) $-\dfrac{58}{5}$ (B) $\dfrac{29}{2}$ (C) 11   (D) 15   (E) $-\dfrac{22}{5}$

13. A spherical balloon has volume 240π in^3. If 20% more air is blown into the balloon (and it still retains its spherical shape), find the radius of the enlarged balloon.

13. _____

(A) $2\sqrt[3]{9}$ in.   (B) 6 in.   (C) $6\sqrt{2}$ in.   (D) $\sqrt[3]{36}$ in.   (E) $\dfrac{6\sqrt[3]{180}}{5}$ in.

14. Find the area of a hexagon with apothem 6 inches.

14. _____

(A) $36\sqrt{3}$ in^2 (B) 72 in^2 (C) $72\sqrt{3}$ in^2 (D) $216\sqrt{3}$ in^2 (E) 144 in^2

15. Below is a net that can be used to create a solid, formed by three congruent rectangles and two congruent equilateral triangles. How many vertices and edges does the solid have?

(A) 10 vertices and 14 edges
(B) 18 vertices and 18 edges
(C) 6 vertices and 14 edges
(D) 9 vertices and 9 edges
(E) 6 vertices and 9 edges
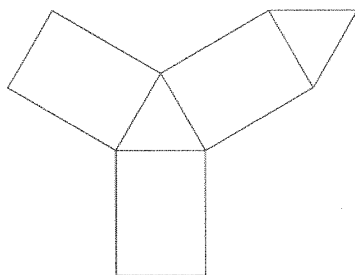
15. _____

16. A construction can be found below with all steps shown except for any final straightedge marking(s). Which of the following constructions is being performed?

(A) Angle bisector
(B) Equilateral triangle
(C) Segment bisector
(D) Line through a point parallel to a given line
(E) Construct a circle given 3 points

16. _____

A   B
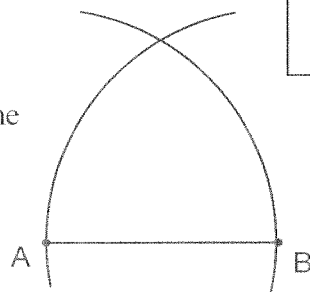
17. In spherical geometry, a line is defined to be a great circle of the surface of a sphere. Two lines are parallel in spherical geometry if they never intersect.
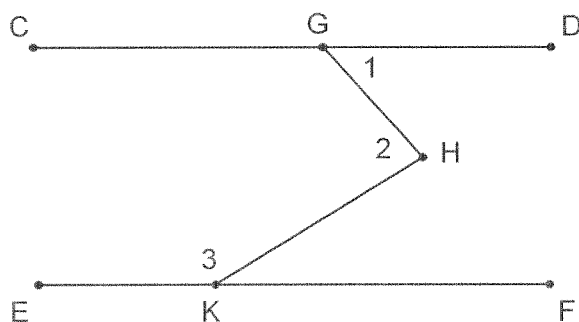
    Given line ℓ on a sphere along with the above definitions, how many lines on the sphere are parallel to line ℓ?

    17. _____

    (A) 0   (B) 1   (C) 2   (D) 4   (E) An infinite number

18. In the diagram below, $\overline{CD} \parallel \overline{EF}$, $m\angle 1 = (x-7)^\circ$, $m\angle 2 = (3x)^\circ$, and $m\angle 3 = (9x+19)^\circ$. Find the value of x.

    18. _____

    (A) $\dfrac{161}{12}$ (B) 14   (C) $\dfrac{84}{5}$ (D) $-\dfrac{3}{2}$ (E) $\dfrac{168}{13}$

19. Given regular octagon ABCDEFGH, remove vertex G and use the remaining vertices to form heptagon ABCDEFH. Find the measure of $\angle F$.
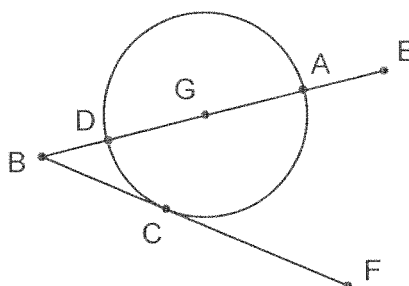
    (A) 45°

    (B) 157.5°

    (C) $128 \frac{4}{7}^\circ$

    (D) 135°

    (E) 112.5°

    19. _____

20. $\overline{BE}$ is a secant of circle G and $\overline{BF}$ is tangent to circle G at point C. If $m\overset{\frown}{AC} = 120°$ and the diameter of circle G is 10 cm, find the length of $\overline{BC}$.

(A) $5\sqrt{3}$ cm
(B) $5\sqrt{2}$ cm
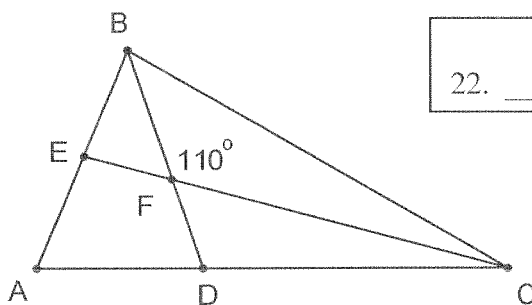(C)   10 cm
(D)   5 cm
(E)   30 cm

20. _____

21. Quadrilateral ABCD is inscribed in a circle, with $m\angle C = 70°$. Find the measure of $\angle A$.

21. _____

(A) $70°$ (B) $110°$ (C) $145°$ (D) $140°$ (E) Can not be determined

22. $\overline{CE}$ and $\overline{BD}$ are angle bisectors of $\triangle ABC$ which intersect at point F. If $m\angle BFC = 110°$, find the measure of $\angle A$.
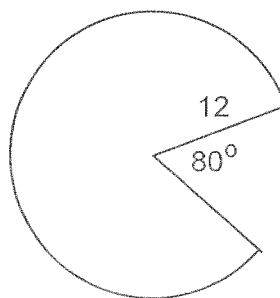
(A) $40°$
(B) $55°$
(C) $70°$
(D) $110°$
(E)   Cannot be determined

22. _____

23. A sector with central angle $80°$ and radius 12 has been removed from a figure, as shown below. Find the area of the portion of the circle which remains.

(A) $\dfrac{56\pi}{3}$
(B) $\dfrac{56\pi + 72}{3}$
(C) 32π
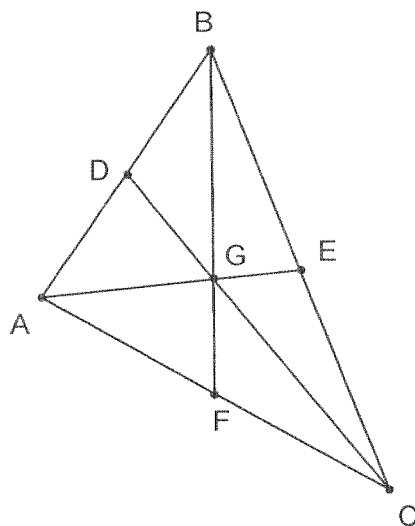(D) 128π
(E) 112π

23. _____

24. Triangle ABC has medians $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ which intersect in point G as shown below. Given that $AF = DB$, $BC = 10$, and $FG = \sqrt{11}$, find the length of $\overline{AE}$.

(A) $4\sqrt{6}$

(B) $5\sqrt{3}$

(C) $3\sqrt{11}$

(D) $\sqrt{21}$

(E) $3\sqrt{19}$

24. _____

25. Circle D is inscribed in right triangle ABC with right angle C, and $\overline{AB}$ is tangent to the circle at G. If $CB = 12$ and $GB = 7$, find AC.

(A) 23

(B) 27

(C) 30

(D) 35

(E) 37
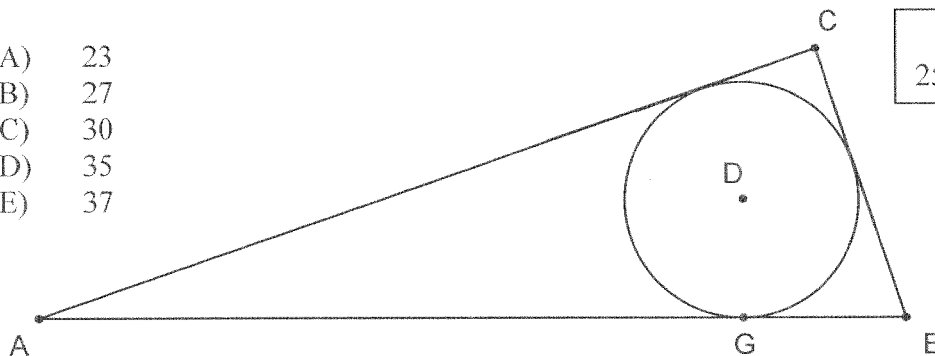
25. _____

26. Two pyramids are similar. The lateral area of the first pyramid is 240 cm^2, and the lateral area of the second pyramid is 540 cm^2. If the larger pyramid has a volume of 810 cm^3, find the volume of the smaller pyramid.

(A) $\dfrac{640}{9}$ cm^3

(B) 540 cm^3

(C) 360 cm^3

(D) 720 cm^3

(E) 240 cm^3

26. _____

27. A company manufactures dice in large quantities and ships them to other game companies. The manufacturer gets a cube-shaped cardboard box ready to send to a customer, and fills it with 512 dice. If the dice are stacked neatly in rows in such a way that they fill the entire volume of the box, how many dice are touching at least one face of the cardboard box?

(A) 296
(B) 216
(C) 256
(D) 384
(E) 512

27. _____

28. A cube is shown in Figure 1 below. In Figure 2, vertices A, B, C, and D of the same cube are shown as vertices of a right triangular pyramid. If the volume of the cube is 54 cm^3, find the volume of the right triangular pyramid.

(A) 18 cm^3
(B) 12 cm^3
(C) 36 cm^3
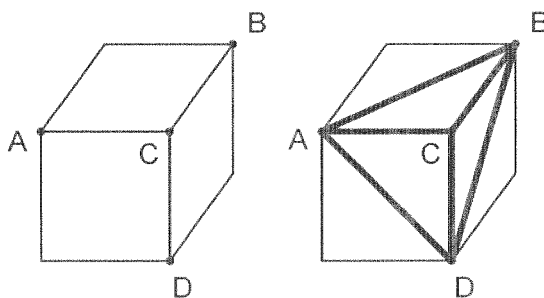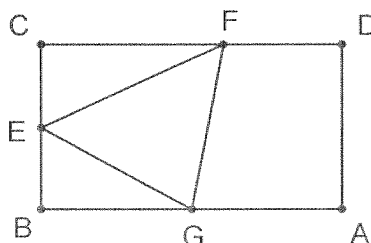(D) 9 cm^3
(E) 27 cm^3

28. _____

Figure 1

Figure 2

29. Rectangle ABCD is shown below with $CD = 10$ and $AD = 8$. If E is the midpoint of $\overline{BC}$, G is the midpoint of $\overline{AB}$, F is a point on $\overline{CD}$, and the area of $\triangle EFG$ is 18, find the length of $\overline{DF}$.

(A) 8
(B) 7
(C) 4
(D) 5
(E) 6

29. _____

30. A building is made from stacked cubes, and views from three different perspectives are illustrated below. *There are exactly ten stacks that are one unit high.*
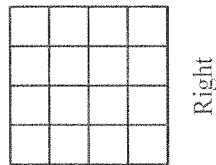
Top View: Front View: Right View:

Front
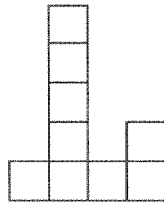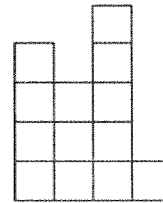
How many cubes are needed to construct this building?

30. _____

(A) 22   (B) 24   (C) 25   (D) 28   (E) 33

31. Which of the following polygons will tessellate the Euclidean plane?

I.   Every pentagon
II. Every triangle
III. Every hexagon
IV. Every quadrilateral

31. _____

(A)   II only
(B)   III only
(C)   IV only
(D)   II and IV only
(E)   None of the above

32. Circle A and Circle B have common internal tangent $\overline{FG}$ which intersects $\overline{AB}$ at E. If circle A has radius 10, circle B has radius 8, and $AB = 27$, find the length of $\overline{BE}$.

(A)   4
(B)   8
(C)   9
(D)   10
(E)   12

32. _____
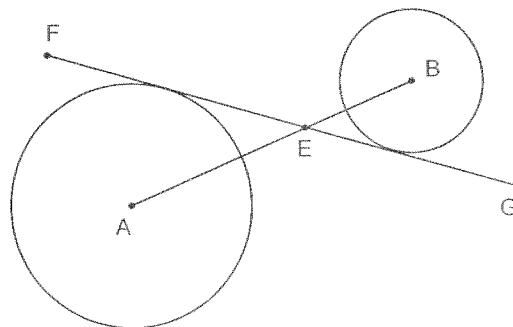
33. A square pyramid is made of foam and has base edge 12 cm and height 8 cm. A knife slices through the apex perpendicular to the base and through two opposite vertices of the base (cutting along one of the diagonals of the base), forming two distinct congruent polyhedra. Find the total surface area of one of the resulting polyhedra.

(A) 240 cm^2

(B) $\left(312 + 96\sqrt{2}\right) \text{ cm}^2$

(C) $\left(192 + 24\sqrt{17}\right) \text{ cm}^2$

(D) $\left(264 + 60\sqrt{2}\right) \text{ cm}^2$

(E) $\left(192 + 48\sqrt{2}\right) \text{ cm}^2$

33. _____

34. Triangle ABC exists on a coordinate plane, and the midpoints of its sides are located at the points $(2,1)$, $(3,7)$, and $(5,4)$. Find the area of triangle ABC.

(A) 10   (B) 15   (C) 20   (D) 30   (E) 36

34. _____

35. In the diagram below, an equilateral triangle with side length 4 inches is shown with both its inscribed and circumscribed circles. Find the area of the gray shaded region.

35. _____

(A) $\dfrac{4\pi}{3} \text{ in}^2$

(B) $\left(\dfrac{8\pi}{9} + \dfrac{4\sqrt{3}}{3}\right) \text{ in}^2$

(C) $\left(\dfrac{16\pi}{9} + \dfrac{2\sqrt{3}}{3}\right) \text{ in}^2$

(D) $\left(\dfrac{8\sqrt{3}}{3} - \dfrac{8\pi}{9}\right) \text{ in}^2$

(E) $\dfrac{8\pi\sqrt{3}}{3} \text{ in}^2$
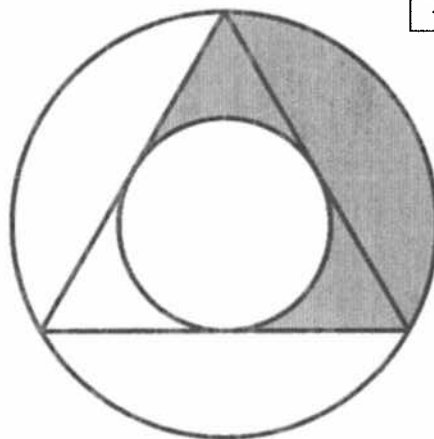
END OF EXAM ☺
(Make sure all answer blanks are filled in with the letter of the correct answer.)

Algebra II Exam
2009 University of Houston Math Contest

Name: _____

School: _____

Please read the questions carefully and give a clear indication of your answer on each question.

There is no penalty for guessing.

Judges will use written comments and/or calculations to settle ties.
Good luck.

University of Houston
2009 High School Math Contest
Algebra II Exam

Name_____

School_____

Exam Time : 1 hour

No calculator allowed.

Write the letter (a, b, c, d or e) corresponding to your answer in the table below.

There is no penalty for guessing.

In the event of a tie, students' work will be used to determine the winner – so show your work clearly on your test or scratch paper.

Problem	Answer	Problem	Answer
1		14	
2		15	
3		16	
4		17	
5		18	
6		19	
7		20	
8		21	
9		22	
10		23	
11		24	
12		25	
13		26	

1. Which of the following are solutions to the equation $x^{2010} + 1 = 0$?

    I. 1   II. -1 III. i IV. $-i$

    (a) I and II

    (b) II

    (c) III

    (d) III and IV

    (e) II and III

2. John is the scheduler for his town's softball league. There are 8 teams in the league. In the regular season, each team plays all the other teams twice, and there is only one game per evening. John also reserves the softball field for 3 extra evenings, in case there are games cancelled due to weather. How many evenings does John need to reserve on the field's calendar for the regular season of the softball league?

    (a) 28

    (b) 56

    (c) 31

    (d) 59

    (e) 53

3. $\log_4 x = -\dfrac{3}{2}$ Solve for x.

    (a) -6

    (b) $\dfrac{1}{8}$

    (c) 8

    (d) $\dfrac{1}{6}$

    (e) $\dfrac{1}{64}$

4. For which of the following bases, b, is the number 121_b a perfect square?

    (a) 4

    (b) 5

    (c) 3

    (d) all of the above

    (e) none of the above

5. Simplify

$$\frac{2x^2 - 5x + 3}{3x^2 + x - 4} \times \left(\frac{2x^2 + 3x - 9}{3x^2 + 13x + 12} \right)^{-\frac{1}{2}}$$

(a) $\sqrt{\dfrac{2x-3}{3x+4}}$

(b) $\sqrt{\dfrac{3x+4}{2x-3}}$

(c) $\sqrt{\dfrac{2x+3}{3x-4}}$

(d) $\sqrt{\dfrac{3x-4}{2x+3}}$

(e) $\sqrt{\dfrac{x-1}{x+3}}$

6. The Puzzleberry Citrus Orchards runs an unusual end of the season sale every year. The bags they give out hold just 7 oranges each. The orchard then charges $0.25 for each bag of seven oranges, and $0.75 for each additional single orange. According to this system, which of the following would cost the most?

    (a) 11 oranges
    (b) 32 oranges
    (c) 50 oranges
    (d) 57 oranges
    (e) 70 oranges

7. To keep their profits high without raising the price of a can of beans, the Keen Bean Company is planning on reducing the size of their cans. They start with a can of height h inches and radius r inches, which has volume $V = \pi r^2 h$ cubic inches. Which of the following will result in a can with the smallest volume?

    (a) Keep h the same and reduce r by 20%
    (b) Keep r the same and reduce h by 20%
    (c) Reduce h by 10% and reduce r by 10%
    (d) Increase h by 10% and reduce r by 30%
    (e) All of the above will have the same volume.

8. Consider the function f(x) such that $f(mn) = f(m-n)$ for all real numbers m and n. If $f(4) = 3$, find $f(-2) + f(6)$.

    (a) 0

    (b) 6

    (c) 3

    (d) 2

    (e) It cannot be determined with the information given.

9. Let x and y be real numbers such that $2^x = 24$ and $24^y = 32$. Find xy.

    (a) 16

    (b) 8

    (c) 5

    (d) 4

    (e) 9

10. Let a, b and c be real numbers, and let $P(x) = ax^9 + bx^5 + cx + 3$. If $P(-5) = 17$, find $P(5)$.

    (a) -17

    (b) -11

    (c) 14

    (d) 17

    (e) It cannot be determined from the information given.

11. Bob noticed that right now his dad, Steve, is five times older than his age. Last year, Bob's age was half that of his brother Bill. Nine years from now, Steve will be twice as old as Bill. What is the sum of the current ages of Bob, Bill and Steve?

    (a) 55

    (b) 33

    (c) 35

    (d) 64

    (e) 65

12. Find the vertex of the parabola given by the equation $f(x) = 3x^2 - 18x - 29$.

    (a) $(9, -110)$

    (b) $(-3, -56)$

    (c) $(3, -38)$

    (d) $(-3, -2)$

    (e) $(3, -56)$

13. Find the equation of the line which passes through the center of both of the following circles:

$$(x+3)^2 + (y-3)^2 = 9$$
$$(x-1)^2 + (y+2)^2 = 16$$

    (a) $5x + 4y = -3$
    (b) $4x + 5y = 3$
    (c) $5x + 4y = 3$
    (d) $x + 2y = 5$
    (e) $x - 2y = 5$

14. Let h and k be the roots of the equation $2x^2 - 9x + c = 0$. If $4hk = 11$, find $h + k + c$.

    (a) $\dfrac{47}{8}$
    (b) 20
    (c) 1
    (d) 10
    (e) 11

15. A wire which is x inches long is bent into the shape of a rectangle. If the length of the rectangle is 2 inches more than three times its width, what is the area of the rectangle?

    (a) $3x^2 + 2x$
    (b) $\dfrac{3}{64}x^2 - \dfrac{1}{8}x - \dfrac{1}{4}$
    (c) x^2
    (d) $\dfrac{3}{8}x^2 - x - 2$
    (e) $3x^2 - 8x + 16$

16. Let $f(x) = \dfrac{3x-4}{2x+1}$. Find its inverse $f^{-1}(x)$.

    (a) $\dfrac{2x+4}{3x-1}$
    (b) $\dfrac{-x-4}{2x}$
    (c) $\dfrac{x+4}{3-2x}$
    (d) $\dfrac{x+4}{2x-3}$
    (e) $\dfrac{2x+1}{3x-4}$

17. Solve $\sqrt{7+\sqrt{x+\sqrt{x+2}}} = 3$.
   (a) 2
   (b) 4
   (c) 14
   (d) -2
   (e) 7

18. Let a, b and c be nonzero digits (1-9). Let abc, bca and cab be the three digit numbers obtained by placing a, b and c next to each other. Which of the following is NOT always true about the sum $S = abc + bca + cab$
   (a) S is divisible by 3
   (b) S is divisible by 2
   (c) S > 330
   (d) S is divisible by 37
   (e) $\dfrac{S}{(a+b+c)}$ is an integer

19. Solve $|x+3|+|x-5| \le 10$.
   (a) [2,6]
   (b) $(-\infty,-4)\cup(6,\infty)$
   (c) [−4,6]
   (d) [−13,15]
   (e) $(-\infty,-3)\cup(5,\infty)$

20. Find the area of the region in the first quadrant which is the solution set to the following system of linear inequalities:
$$3x + y \le 15$$
$$y \ge 2x$$
$$x \ge 0$$
$$y \ge 0$$

   (a)   45
   (b)   15
   (c)   30
   (d) $\dfrac{45}{2}$
   (e) $\dfrac{75}{2}$

21. Hal P. Miself's two hour self-help seminar was so bad that 40% of his audience left after the first 30 minutes. By the end of the one hour coffee break, 50% of those remaining after the first half hour left. When Hal stopped to look for something in his briefcase after 1 ½ hours of the seminar, 60% of the people remaining after the coffee break left, and the rest stayed until the end of the seminar. There were only 30 people in the room to hear Hal's one good piece of advice at the end of his talk. How many people were in the audience when Hal started speaking?

(a) 100
(b) 360
(c) 250
(d) 375
(e) 500

22. One jelly bean is drawn at random from a jar containing red and black jelly beans. If there were 2 more black jelly beans in the jar, the probability of drawing a black jelly bean would be 1/3. If there were 2 less black jelly beans in the jar, the probability of drawing a black one would be ¼. How many red jelly beans are actually in the jar?

(a) 34
(b) 24
(c) 14
(d) 48
(e) 10

23. Which of the following is a solution to the equation $x^2 + \sqrt{12}\, x - 28 = 0$?

(a) $-\sqrt{3} - \sqrt{31}$
(b) -14
(c) $-\sqrt{3} + 5i$
(d) $-6 + \sqrt{31}$
(e) $-6 + 5i$

24. Which of the following is a true statement for all nonzero real numbers x and y satisfying the inequality $\dfrac{1}{x} - \dfrac{1}{y} > 0$?

(a) $x > y$
(b) $y > x$
(c) $\dfrac{1}{x} + \dfrac{1}{y} > 0$
(d) $\dfrac{x+1}{x} - \dfrac{y+1}{y} > 1$
(e) $\dfrac{x}{xy} < \dfrac{y}{xy}$

25. Emma has a collection of wooden blocks. She has three sizes of blocks: large blocks are cubes with 4 inch sides, medium blocks are cubes with 3 inch sides, and small blocks are cubes with 2 inch sides. Emma has three times as many small blocks as large blocks. She also has two more small blocks than medium blocks. Her block collection fits perfectly into a $8in. \times 10in. \times 12in.$ box when she picks them all up. If Emma were to stack all of her blocks into a vertical tower one block wide, how tall would this tower be?

    (a) 9 feet

    (b) 40 feet

    (c) 108 feet

    (d) 8 feet

    (e) 7 feet and 5 inches

26. Given that $\left(2^{a}\right)^{b} = 2^{a} \times 2^{b}$, find the value of a if $b = 3$.

    (a) $\dfrac{2}{3}$

    (b) $\dfrac{3}{2}$

    (c) 0

    (d) 1

    (e) a can be any real number